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MARINE PHYSICS

Mathematical Modeling of Turbulent Jets of Deep-Water Sewage Discharge into Coastal Basins

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Abstract—The basic properties of the dynamic model of a turbulent jet formed by a deep-water sewage discharge into the stratified environment of coastal regions are considered. The model developed was used to estimate the parameters of a floating-up jet of deep wastewater discharge from Sand Island into the basin of Mamala Bay (Hawaii) depending on the season and discharge operation mode. The estimates of the float-up depths of the jet and the initial dilution of the jet were estimated on the basis of model calculations using experimental data on the vertical profiles of the water temperature and salinity under the actual conditions of stratification in the study region. It is shown that the further propagation of the wastewater jet depends on tidal events and internal waves generated by tides. The appearance of turbulent jets at the sea surface was recorded. The model estimates of the parameters of the wastewater discharge were compared with the results of experimental measurements. Good agreement was found, which indicates that the physical mechanisms of the propagation of turbulent jets in a stratified medium are adequately described by the model.

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INTRODUCTION

The anthropogenic impact on the waters of the World Ocean, which increases every year, negatively influences the ocean ecosystems and, first of all, the coastal regions [1, 7–10, 12]. One of the strongest sources of pollution in these regions is the discharge of sewage waters. Therefore, in the last years, the study of this phenomenon has attracted more and more attention [9, 10, 12].

The study of the processes related to the influence of deep-water discharges on coastal basins is carried out by means of in situ hydrophysical measurements using contact sensors [9, 14, 15, 17, 19], aerospace methods [1, 3–5, 14, 16, 20], and by applying the methods of mathematical modeling of the impact of a jet on the water medium [1, 3, 6, 11, 13].

Contact in situ measurements are carried out either from hydrophysical research vessels or using moored stations deployed at different places of the basin studied. Hydrophysical measurements make it possible to obtain the actual pattern of the pollution in the coastal zones; however, they are labor consuming, require large costs, and do not make possible research over large basins.

In the last years, remote aerospace methods [1–5, 14, 16, 20] have been applied wider and wider in the monitoring of anthropogenic influences on the coastal basins. The aerospace methods make possible to monitor large basins with the required periodicity over a long period of time. This allows us to analyze the spatial, temporal, and seasonal dependences of the characteristics of the phenomena under study [1, 14].

Using the methods of mathematical modeling, one can estimate the quantitative characteristics of the variations in the parameters of the water medium including those occurring under anthropogenic influence in different hydrometeorological conditions and perform a forecast of the negative impacts of these processes on the ecological state of coastal basins [1–3, 6, 9, 11, 13].

It was mentioned above that deep-water sewage discharges have the strongest anthropogenic influence on the ecosystems of the coastal regions. Wastewater discharge is carried out through deep pipelines to large depths at a sufficient distance from the coast to decrease the negative influence on the aquatic medium [9]. Diffusers of discharge devices are located below the density jump since the latter hampers upwelling of wastewaters to the surface. Meanwhile, the density stratification of seawater is subjected to significant daily and seasonal fluctuations. If the density decreases, situations can occur when the waters discharged would rise to the surface and negatively influence the coastal regions.

It is noteworthy that, in the processes of propagation of deep discharges, selected particular features of the coastal regions such as tidal phenomena and upwelling (upward water motion related to the offshore flow of the surface water layer from the coastal zone to the open sea due to the Ekman transport) play important roles.

The pollution of the coastal zones of the sea caused by the discharge of sewage waters, which increases

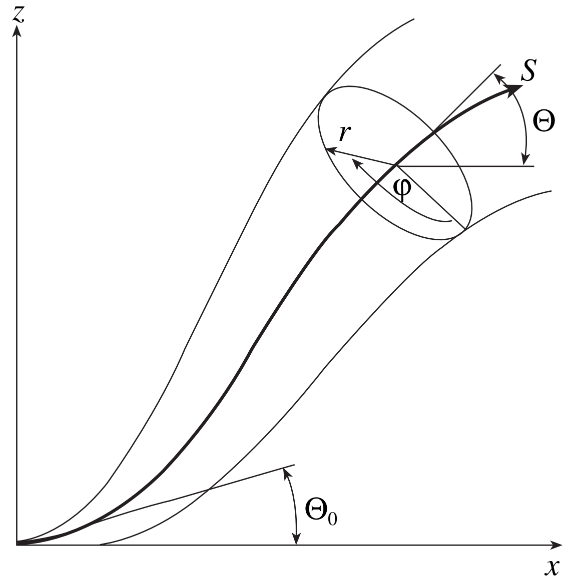

Fig. 1. Geometry of a floating-up jet in a stably stratified fluid.

every year, forces us to find methods for minimizing these negative processes. It is natural that the most radical method of solving this problem is the complete termination of discharge of sewage waters or their preliminary purifying to admissible concentrations of harmful substances. However, the measures necessary for these purposes require high costs. Therefore, their realization in the nearest future is hardly possible. In this relation, it is necessary to use other methods and possibilities to decrease the negative impacts of deep-water discharges in coastal regions.

Effective methods of monitoring and application of prognostic mathematic models, which make it possible to estimate the behavior of jets under different hydrometeorological conditions and to forecast the appearance of most unfavorable and favorable situations for discharges, play an important role in the solution of this problem. This facilitates effective actions for decreasing the level of negative anthropogenic influence on coastal regions.

Let us consider in detail the potentialities of the methods of mathematical modeling for investigating the influence of deep discharges on coastal regions.

MAIN PROVISIONS OF THE MATHEMATICAL MODEL FOR THE PROPAGATION OF JETS IN A STRATIFIED MEDIUM

Burying of the discharged waters in the sea is usually performed through sewage outfalls located in the bottom layer, which may have different constructions. However, all of them generate turbulent jets in the seawater, which usually differ in density from the surrounding medium [3, 11, 13]. Thus, the discharged non-salty waters would float up.

Let us choose the following parameters as the initial ones for modeling the turbulent jet: the water discharge Q, which is determined as the water volume passing through the transversal section of the jet in a of unit time; the momentum M equal to the product of the discharge rate of the jet by its velocity; and the density difference between the waters in the jet and in the sea at the level of discharge $\Delta\rho$, which determines the buoyancy of the jet together with the discharge [3]

$$F = g\Delta\rho Q/\rho_0,$$

where g is the acceleration due to gravity and ρ_0 is the density of the medium at the level of discharge.

In the course of the turbulent jet propagation, its discharge increases due to the entrainment of the surrounding water into the motion. If the medium is homogeneous by density and the difference between the densities of the medium and the jet is zero, its momentum remains constant during the spreading of the jet. In the opposite case, only the horizontal component of the momentum is conserved, whereas the vertical component changes under the influence of buoyancy forces.

In the case when the medium is homogeneous by density but its density is not equal to the density of the jet, the buoyancy reserve is conserved (if the equation of state of the water is linear) and, potentially, the jet floats up infinitely [3]. If we consider a fluid (Lagrangian) element of the floating-up jet in a stably stratified medium, then, owing to the mixing with the surrounding waters, its density would increase while the density of the surrounding water decreases (since the element displaces upwards). Thus, at a certain level, the difference of the densities would turn to zero and the jet would not float up.

In practice, it is important to calculate the level that is reached by the jet when it floats up, the dilution that occurs during the floating up, and the thickness of the layer in which the discharge water is localized after the end of the floating up (Ozmidov scale [13]). Let us use the following model of a floating-up jet to solve this problem [3, 6, 11, 13].

We assume that the turbulent jet is injected into the medium at a depth of z_0 and at an incident angle of Θ_0 with respect to the horizontal plane xz (the z axis is directed upwards). The medium is incompressible and remains at rest. Its density $\rho_a(z)$ depends on the vertical coordinate, and $d\rho_a/dz < 0$; i.e., the medium is stably stratified. We assume that the density of the jet $\rho(z)$ at the level z_0 is smaller than the density of the medium: $\rho(z_0) = \rho_1 < \rho_a(z_0)$. Thus, the jet would float up.

Consideration of the problem is carried out in a coordinate system related to the jet (Fig. 1): s is the coordinate along the jet, r is the radial coordinate, and φ is the angle between the plane xz and the given radius vector. It is assumed that the profiles of the velocity u^* in transversal sections of the jet, as well as the profiles of the density differences between the medium and the jet ($\rho_a^* - \rho$), are similar and described by the Gaussian curves

$$u^*(s, r, \varphi) = u(s)\exp(-r^2/b^2), \quad (1)$$

$$\frac{\rho_a^*(s, r, \varphi) - \rho^*(s, r, \varphi)}{\rho_0} = \frac{\rho_a(s) - \rho(s)}{\rho_0}\exp\left[-\frac{r^2}{\lambda^2 b^2}\right], \quad (2)$$

where $u(s)$ and $\rho(s)$ are the velocity and density of the jet, $\rho_0 = \rho_a(0)$ is the reference density, $b = b(s)$ is the characteristic half-width of the jet, and $\lambda = 1.16$ is the constant describing the experimental fact that the density difference profiles in the jet are more diffused than the velocity profile. Assumptions (1) and (2) about the axial symmetry of the jet propagation in a stably stratified medium are admissible only for the initial (active) phase [3].

The spreading of the jet can be described by an equation system that is obtained by integration of the equations of the continuity, motion, and heat and salt balances. Integration is performed over the transversal section of the jet or under assumptions about self-similar properties of the velocity profiles of the form (1) with the temperature and salinity profiles being similar to (2). The application of the equations of the heat and salt balances is related to the necessity of taking into account the nonlinearity of the equation of the state of the seawater. If we ignore it, this can result in a significant distortion of the results, and, in the case of low temperatures and low salinity in the basin, this can lead to principal errors (for example, forecast of a floating-up jet can be made instead of a deepening jet [11].

The equation system is written as:

$$\frac{d}{ds}(ub^2) = 2\alpha ub, \quad (3)$$

$$\frac{d}{ds}(u^2b^2\cos\Theta) = 0, \quad (4)$$

$$\frac{d}{ds}(u^2b^2\sin\Theta) = 2g\lambda^2b^2\frac{\rho_a - \rho_0}{\rho_0}, \quad (5)$$

$$\frac{d}{ds}[ub^2(T_a - T)] = \frac{1+\lambda^2}{\lambda^2}b^2u\frac{dT_a}{ds}, \quad (6)$$

$$\frac{d}{ds}[ub^2(S_a - S)] = \frac{1+\lambda^2}{\lambda^2}b^2u\frac{dS_a}{ds}, \quad (7)$$

where $T_a(s)$ and $S_a(s)$ are the temperature and salinity of the medium, respectively; $T(s)$ and $S(s)$ are the temperature and salinity of the jet, respectively; and $\alpha = 0.057$ is the entrainment coefficient.

System (3)–(7) is supplemented with two geometrical relations between the coordinates related to the jet s and Θ and the Cartesian coordinates x and z, as well as with the equation of state of the seawater

$$\frac{dx}{ds} = \cos\Theta, \quad (8)$$

$$\frac{dz}{ds} = \sin\Theta, \quad (9)$$

$$\rho = \rho(T, S). \quad (10)$$

The equation of state relates the temperature and salinity to the density and does not include the pressure because we are analyzing the potential density.

Thus, we obtain a closed system of seven ordinary differential equations (3)–(9) and the equation of state of the seawater (10) with eight dependent variables (u, b, T, S, ρ, x, y, and Θ) and one independent variable s.

This system can be supplemented by an equation for the mean time t of the propagation of a fluid element along the trajectory of the jet:

$$dt = \frac{ds}{\bar{u}} = \frac{2ds}{u}, \quad (11)$$

where the mean velocity is determined from the condition that the Gaussian distribution of the velocity (1) is substituted with a constant velocity $\bar{u} = u/2$ in the section of the jet with a radius $\bar{b} = \sqrt{2b}$ at constant discharge and momentum.

If the equation of state of the seawater is assumed to be linear and the vertical gradients of the temperature and salinity are constant, then, for system (3)–(11), it is possible to analytically obtain integrals (5) and (10), which relate the buoyancy flux $b^2ug(\rho_a - \rho)/\rho_0$ to the vertical component of the momentum flux $u^2b^2\sin\Theta$ (5) and the time t (11).

It follows from the latter that the time of floating up of the jet is equal to half of the Brunt–Vaisala period:

$$t_0 = T/2 = \pi\left(\frac{g}{\rho_0}\frac{\partial\rho}{\partial z}\right)^{-1/2}. \quad (12)$$

After the end of the floating up, the liquid element of the jet oscillates vertically near the equilibrium depth and the oscillations decay. It is possible to analytically integrate system (3)–(11) only in the quasi-horizontal approximation (when the relation $\Theta \ll 1$ is true during the process of floating up).

The following initial conditions are specified for the solution of equation system (3)–(11):

$$u = u(0), \quad b = b(0), \quad T = T(0), \quad S = S(0), \quad \Theta = \Theta_0, \quad x = 0, \quad z = z_0, \quad t = 0, \quad (13)$$

where $u(0)$, $b(0)$, $T(0)$, and $S(0)$ are the initial values of the parameters of Gaussian distributions (1) and (2). These initial conditions are determined from the initial values of the transport Q_0, the nominal discharge velocity u_0, the temperature T_0, and the salinity S_0 of the jet calculated from the relations

$$u(0) = 2u_0,$$
$$b(0) = \sqrt{\frac{Q_0}{2\pi u_0}},$$
$$\frac{T_a(z_0) - T(0)}{T_a(z_0) - T_0} = \frac{1+\lambda^2}{\lambda^2}, \qquad (14)$$
$$\frac{S_a(z_0) - S(0)}{S_a(z_0) - S_0} = \frac{1+\lambda^2}{\lambda^2}.$$

In addition, we specify the vertical profiles of the temperature $T_a(z)$ and salinity $S_a(z)$ in the basin.

System of ordinary equations (3)–(11) at initial conditions (13) and (14) can be solved numerically using Euler or Runge–Kutta methods. Since the jet propagates in a stably stratified medium, its vertical size cannot grow infinitely, because this requires an infinitely large energy for performing work against the buoyancy forces. Hence, the jet would be localized in a layer with the width h_*. The problem of estimating the thickness of the layer h_* is similar to the problem of determining the so-called Ozmidov scale, the maximal size of the turbulent eddies that can exist in a medium with stable stratification [13].

The relation for estimating the thickness of the layer in which the jet propagates can be obtained from the dimension considerations. If the jet propagates quasi-horizontally, its momentum is approximately constant: $M(s) \approx M_0 = Q_0u_0 = \text{const}$.

In turn, the stratification of the medium is determined by the Brunt–Vaisala frequency $N = [-(g/\rho_0)\partial\rho_a/\partial z]^{1/2}$. Thus, we can suppose that the thickness of the layer of the jet propagation h_* is determined by the parameters M_0 and N, and it can be obtained from the considerations of the dimensions

$$h_* = cM_0^{1/4}N^{-1/2}, \qquad (15)$$

where c is a constant of the order of unity.

Relation (15) was successfully used to describe the jet of sewage discharges to Lake Baikal from the Baikal cellulose factory [6].

In the above considerations, the effect of floating up of the jet was not taken into account. This means that the obtained estimates of the maximal thickness of the jet can be applied to a floating-up jet only under the condition that it was injected horizontally and that, in the course of floating up, it remains horizontal. The condition of a quasi-horizontal propagation of the jet was obtained in [11]. It is written as

$$Nu_0 \gg g\frac{|\Delta\rho|}{\rho_0}, \qquad (16)$$

where $\Delta\rho$ is the initial density difference between the jet and the medium. In the opposite case, when in the course of floating up the vertical component of the momentum reaches values much greater than the horizontal gradient ($Nu_0 \gg g|\Delta\rho|/\rho_0$), the determining factor is the initial buoyancy flux $F_0 = Q_0g|\Delta\rho/\rho_0|$.

We can suggest a relation for the maximal thickness of the jet from the considerations of the dimensions:

$$h_* = c_1F_0^{1/4}N^{-3/4}, \qquad (17)$$

where c_1 is another constant of the order of unity. It should be noted that we know no cases of application of relation (17) for the description of jets in a stratified medium.

Thus, by the end of the active phase, floating up of the jet stops, and its further propagation occurs approximately at the same depth. The jet oscillates near the depth that it has reached by the end of the active phase, and the oscillations decay [3]. The further spreading of the jet in the medium occurs under the action of turbulent diffusion and currents existing in the coastal zone [3]. The passive phase of the jet is characterized by the following particular features [3]:

—The turbulent entrainment stops.

—The horizontal size increases, while the increase in the vertical size stops.

—The thermohaline instability can develop at the lateral boundaries (intrusive interleaving).

—The convective processes of double diffusion (salt fingers) can develop at the upper and lower boundaries, which leads to a significant increase in the vertical volume.

—The admixture is transported by the currents and scatters under the influence of horizontal diffusion.

—The suspended matter is sorted.

A program code was developed for calculating characteristics of the floating-up jet using the model described above.

RESULTS OF CALCULATION OF THE CHARACTERISTICS OF FLOATING-UP JETS

The characteristics of floating-up jets were calculated using a computer code that realized the theory described above for the actual outfall for sewage waters on Sand Island (Honolulu, Hawaii Islands), for which we had data about the parameters of the discharge and the characteristics of the marine environment [18, 21]. The discharge system provides improved primary processing of 70 mln. gallons (31.8 th. m^3) of discharge water per day [18]. The full length of the oceanic part of the pipeline to the diffuser is 3816 m. The internal diameter of the collector is 213 cm. The diffuser through which the discharge waters flow to the basin of Mamala Bay is located at a depth of 69–72 m. The length of the diffuser is 1036 m. It contains 275 outlets with a diameter of 7.8–12.7 cm in the main pipe with a step of 3.66 m in turn on both sides of the pipe. The mean discharge of the sewages is 4.48 m^3/s at a nominal velocity of the flow from the outlets approximately equal to 3 m/s.

Thus, we can assume that the initial input parameters for the single jet model are the following: the discharge is $Q_0 = 4.48/275$ $m^3/s = 0.0163$ m^3/s, and the velocity of the flow is $u_0 = 3$ m/s. The temperature of the discharge waters fluctuates within 23.3–27.3°C. In the model calculations, we assume that $T_0 = 25$°C. The ocean temperature at the diffuser depth fluctuates within 21…24°C.

Since the polluted discharge water is fresh water that passed through preliminary treatment, we assume in the model that $S_0 = 0$ ‰. We can expect that the initial density of the jet specified in the model will be underestimated, which will make it possible to obtain the upper levels of the floating up. We specified the depth at which the water is discharged as $z_0 = 70$ m. In the model calculations, we used the vertical density profiles measured in the region of the discharge in different seasons. They are shown in Fig. 2a [18].

The model calculations of the floating-up jet were performed for four versions of the discharge:

Version 1. The discharge is performed in the horizontal direction in the form of noninteracting jets with an axial symmetry with discharges equal to the discharge of each of the 275 holes: $ub^2 = Q_0/\pi n = 0.00519$ m^3/s, where $Q_0 = 4.48$ m^3/s is the mean planned discharge at a nominal velocity of the flow $u_{dis} = 3$ m/s. The momentum of the jet is equal to $u^2b^2 = Q_0u_{dis}/\pi n = 0.00156$ m^4/s^2.

Version 2. The discharge is performed in two opposite horizontal directions from flat jets with discharges $ub = Q_0/2\sqrt{\pi}L = 0.00122$ m^2/s, where $L = 1036$ m is the length of the diffuser. The nominal velocity of the flow is $u_{dis} = 3$ m/s. The momentum of the jet is equal to $u^2b = Q_0u_{dis}/\sqrt{2\pi}L = 0.00519$ m^4/s^2.

Version 3. The discharge is performed in the horizontal direction in the form of a single jet with an axial symmetry and a discharge $Q_0 = 4.64$ m^3/s (average), $ub^2 = Q_{dis}/\pi = 1.478$ m^3/s. The nominal velocity of the flow is $u_{dis} = 3$ m/s. The momentum of the jet is equal to $u^2b^2 = Q_{dis}/\pi = 4.433 m^4/s^2$.

Version 4. The discharge is performed in the horizontal direction in the form of a flat jet with the discharge $ub = Q_0/\sqrt{\pi}L = 0.0244$ m^2/s ($Q_0 = 4.48$ m^3/s). The nominal velocity of the flow is $u_{dis} = 3$ m/s. The momentum of the jet is equal to $u^2b = Q_0u_{dis}/\sqrt{2\pi}L = 0.01035$ m^4/s^2.

The calculated trajectories of the jets for these four versions of discharge to the basin of Mamala Bay in August are shown in Fig. 2b. As seen from Fig. 2b, if the discharge is performed in this period of time in the form of a single horizontal jet from a round outlet (dotted line), the plume formed would reach the surface of the sea even in the summer, when the stratification of the sea is most stable. As was expected, from the point of view of the minimum float-up depth of the plume, the most favorable version is the one of two flat horizontal jets of opposite directions (dashed line), where the jet reaches a level of 46 m. If a flat jet is discharged horizontally from one side with the previous total discharge and momentum, the level of the float up would be equal to 37 m (dashed–dotted line). The level of the float up would be almost the same (38 m) if we independently consider a horizontal jet with axial symmetry from one of the total 275 holes (solid line).

The actual outfall on Sand Island (Honolulu, Hawaii) is a combination of versions 1 and 2. Indeed, since the distance between the outlets from one side of the diffuser is quite large (7.3 m) near the diffuser where the width of the jets is significantly smaller than the distance between the outlets, they propagate independently of each other, thus, representing version 1. It is possible to describe the jets as flat ones only at a distance from the diffuser (version 2). The calculations for versions 1 and 2 do not greatly differ; therefore, further calculations would be performed for a jet with axial symmetry (version 1).

The trajectories of the jet of deep discharge calculated for different seasons from the density profiles shown in Fig. 2a [18] are shown in Fig. 2c. The jet appeared submerged in all the seasons presented. In January, the level of the float up was the closest to the sea surface and was equal only to $z_0 = -12.7$ m.

The trajectory of the jet of discharged water in the basin of Mamala Bay in January is shown in Fig. 3a as an example. The variations in the main parameters of the jet along the trajectory are the following: the discharge Q/Q_0, the temperature difference $T - T_a$, and the density $\rho - \rho_a$. As seen from Fig. 3a, the absolute value of the density difference decreases rapidly and turns to zero before the density reaches the point of the maximal float up; after this, it oscillates in the opposite phase with the trajectory of the jet and the oscillations decay. The dilution of the jet monotonously increases. The rate of increase fluctuates with a wavelength two times smaller than that of the oscillations of the trajectory, being minimal at the extreme points of the trajectory. The temperature difference, as a whole, behaves similarly to the density difference. Some breaks of the curve $T - T_a = f(x)$ can be easily explained if we take into account the particular features of the density profile (and, hence, of the temperature profile) in the sea in January (Fig. 2a).

We should remember that, although the oscillations of the jet trajectory actually take place, this model cannot describe them quantitatively because the condition of the axial symmetry used in the model is not observed after the jet reaches the maximal point of the trajectory.

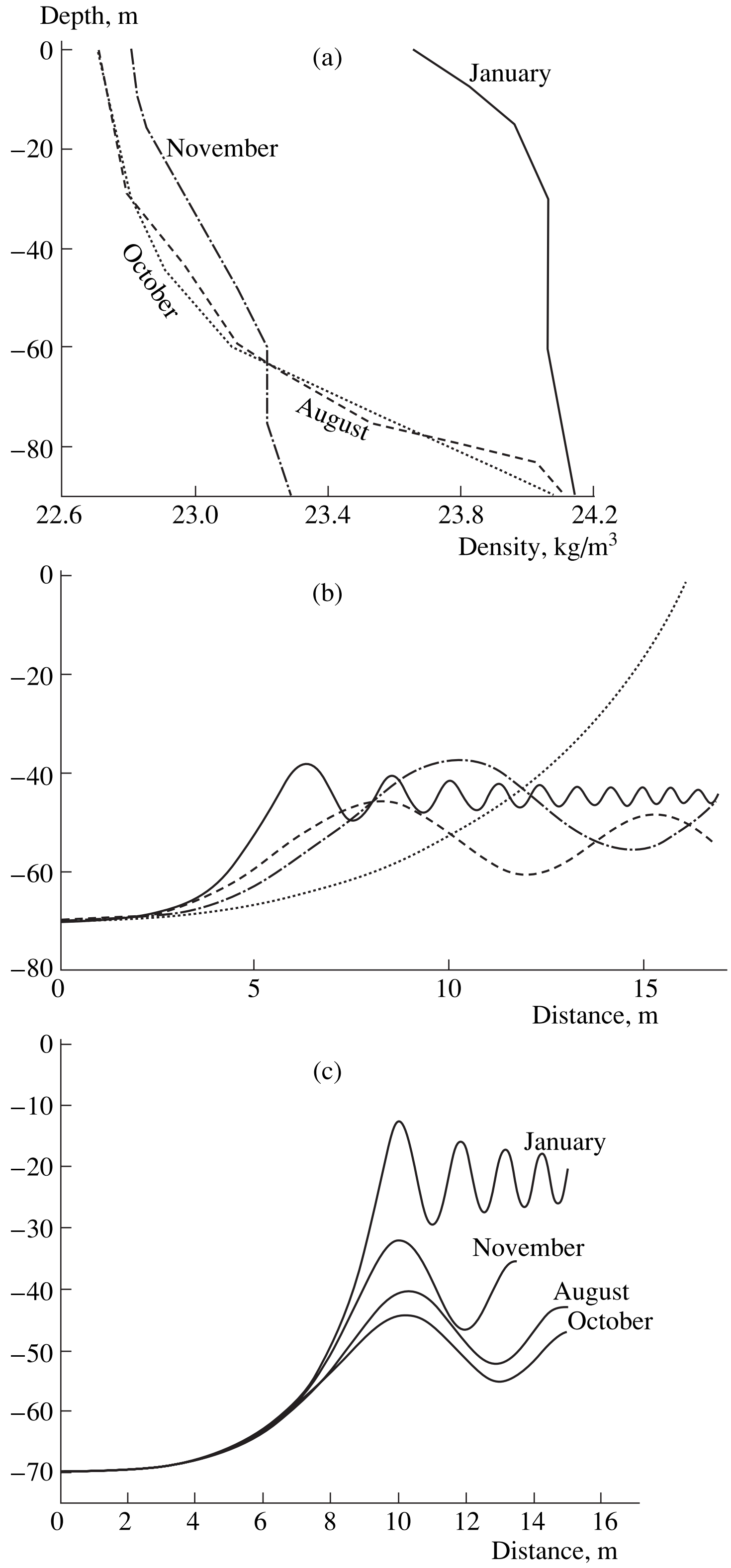

Fig. 2. Parameters of a floating-up jet trajectories in a stably stratified fluid depending on the versions of the discharge and seasons of the year. (a) Vertical profiles of density in the region of discharge in the basin of Mamala Bay (Honolulu, Hawaii) on the basis of measurements in different months [18]. (b) Trajectories of the jets at different versions of the discharge. First version (solid line); second version (dashed line); third version (dotted line); fourth version (dashed–dotted line). (c) Trajectories of the jets of deep discharge in the basin of Mamala Bay (Honolulu, Hawaii) in different seasons of the year.

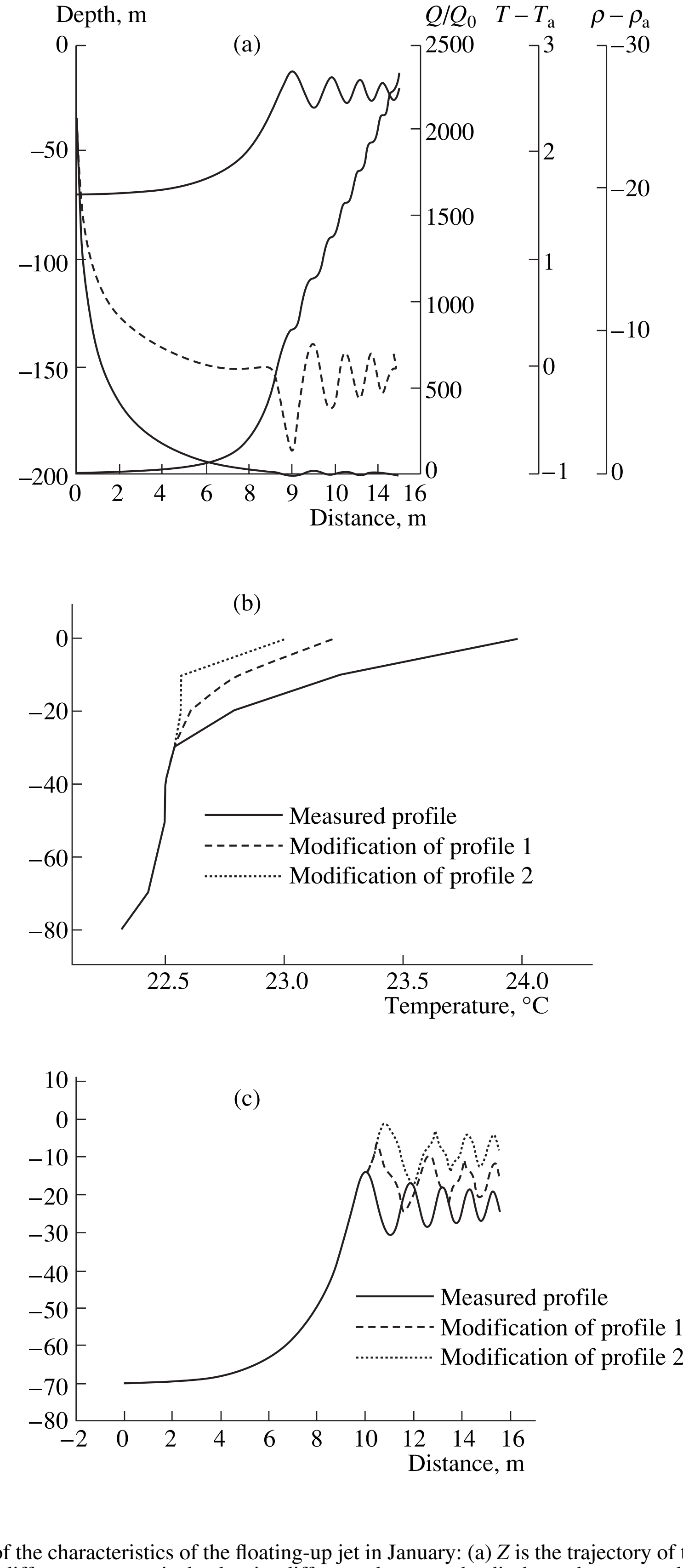

Fig. 3. Model estimates of the characteristics of the floating-up jet in January: (a) Z is the trajectory of the jet, Q/Q_0 is the dilution; $T - T_a$ is the temperature difference; $\rho - \rho_a$ is the density difference between the discharged waters and the surrounding medium in the basin of Mamala Bay; (b) vertical profiles of the seawater temperature in the basin of Mamala Bay; (c) trajectories of the floating-up jets for the temperature distributions with the depth shown in Fig. 2a (c).

Characteristics of discharge jets from the outfall on Sand Island (Honolulu, Hawaii) in different seasons (z_0 is the float-up level, D is the initial dilution, and h_* is the thickness of the layer of propagation)

Parameter	Month			
	January	August	October	November
z_0, m	–12.7	–40.4	–44.4	–32.3
D	836	275	224	413
h_*, m (Eq. (15))	3.9	4.4	5.1	4.9
h_*, m (Eq. (17))	6.2	7.3	9.1	8.5

The information about the level of the float up z_0, the initial diluting $D \equiv Q(z_0)/Q_0$, and the thickness of the layer of jet spreading calculated from relations (15) and (17) for different seasons of the year are given in the table. It follows from the table that the initial dilution is maximal in the winter season ($D = 836$), when the stratification of the water column is minimal and the depth of the float up of the jet is maximal. The minimal dilution (D = 224–274) occurs at the end of the summer–beginning of the autumn, when the stratification of the water column is maximal; hence, the height of the jet float up is minimal.

The calculation performed demonstrates that, at constant parameters of the discharge, the level of the float up is determined by the characteristics of the marine medium stratification (the distribution of the temperature and salinity over the depth). In the winter period of the year, the density gradient, which characterizes the stability of the medium stratification, is minimal. Hence, during this time, conditions can occur at which the stratification is so weak that it does not prevent the float up of the discharge waters to the surface layer. Weakening of the stratification can occur, for example, under the influence of a coastal upwelling, which we discussed earlier.

Model calculations were performed for estimating the depth of the float up of the jet in the case of a weak stratification of the medium in the winter period of the year. With that end in view, the temperature profile experimentally measured in January was modified for depths of 0–20 m and the height of the jet float up was calculated from the modified profile. The salinity of the seawater in the model calculations was assumed to be constant and equal to $S_a(z) = 35‰$.

The measured temperature profile in the basin of Mamala Bay in January and two modifications of this profile are shown in Fig. 3b. The results of the calculation of the depths of the jet float up for these three temperature profiles are shown in Fig. 3c. It is seen that, at weak variations in the profile (the decrease in the temperature gradient is equal to 0.9), the jet floats up to a level of –6 m, while, at slightly greater variation in the profile (the temperature decrease is equal to 0.8), the jet reaches the surface. The estimates given here show that, in the winter months at slight weakening of the density stratification, the jet of discharged water can reach the surface layer.

We also estimated the possible depths of the jet float up in the summer period. The data about the temperature gradients in the basin of Mamala Bay in the region of the discharge given in [18] were used in the calculations.

The graphs of the vertical temperature distributions in the summer period for three values of the temperature gradient (small, medium, and large) are presented in Fig. 4a. The results of the calculation of the level of the jet float up for these three temperature profiles are presented in Fig. 4b. It is seen that, under the conditions of weak stratification of the medium (a small temperature gradient), the jet floats up to a level of –10 m.

The model calculations performed show that, even in the summer months, conditions can occur under which the discharge waters can float up to the surface layers.

The thickness of the layer of the propagation of discharged water depends on the local stratification at the level of the jet float up. The estimates of h_* calculated from relation (17) appeared slightly greater than those obtained from relation (15). In this case, we trust more the estimates calculated from relation (17) because, in our case, the condition $((g|\Delta\rho|/\rho_0)/(Nu_0) \approx 0.1 \ll 1)$ is observed. Therefore, the jet is quasi-vertical, which corresponds well to the forms of the trajectories shown in Fig 2c.

ESTIMATES OF THE PARAMETERS OF THE JET ON THE BASIS OF HYDROPHYSICAL MEASUREMENTS

Multidisciplinary monitoring of the anthropogenic impact of deep waters on the ecosystems of coastal basins [14] was carried out in the basin of Mamala Bay in the region of Sand Island. During this research, a survey of the coastal region was performed using space craft and sea truth measurements of the hydrophysical, biological, and chemical characteristics of the aquatic medium [14, 15, 17, 19]. The results of the analysis of space images were made in [1, 5, 14, 16, 20]. The results of the contact measurements were used as the input data for the calculations and the verification of the model suggested.

It is noteworthy that sufficiently strong tidal currents (the mean value of the ocean level variation is equal to ~0.3 m) are observed in Mamala Bay, which influence different hydrophysical processes including the propagation of the sewage water discharge [15, 21].

The parameters of the medium stratification measured from several moored buoy stations were used in the numerical experiments [14, 15, 17, 19]. The results of the measurements of the temperature and salinity of the seawater at a station located near the diffuser were used for calculating the vertical density profiles in different tidal phases [14, 16, 18]. This made it possible to estimate the influence of the tides and the internal waves generated by them on the parameters of the floating-up jet of the sewage waters.

The time evolution of the seawater temperature distribution over the depth near the diffuser is shown in Fig. 5a. It is seen from this figure that, under the influence of the internal waves of tidal character, periodic temperature variations occurred whose amplitude at depths of 50–60 m reached 3–4°C. At lesser depths, the amplitude of the temperature fluctuations decreased, and, at depths of 35–40 m, it was equal to 1.0–1.5°C [15, 17].

The hourly mean vertical density profiles plotted for eight time moments during the period from 13:00 September 1 to 13:00 September 2, 2002, are shown in Fig. 5b. During this period of research, the intense density jump layer was located at depths of 30–50 m.

The trajectories of propagation of floating-up jets in the mentioned time periods are shown in Fig. 5c. The graphs of the level of the floating-up jet and the density gradients for eight time moments during the period from September 1 to September 2, 2002, are shown in Fig. 6a. It is seen from these figures that, in the period considered, the jet did not rise higher than 36 m, i.e., not higher than the location of the density jump. The density jump with a strong gradient prevented the floating up of the jet closer to the surface.

The maximal depth of the jet float up equal to –36 m (see Fig. 6a) was observed on September 1, 2002, at 20:00. It is seen that, at this time moment, the weakest stratification of the seawater over the studied period was observed (Fig. 6b). Using the model developed, we also obtained estimates of the initial dilution of the sewage water. The graphs of the variation of the dilution Q/Q_0 and the density gradient $\Delta\rho/\Delta z$ for the period of research are shown in Fig. 6b. It is seen from this figure that the weakest stratification of the seawater corresponds to the maximal value of the dilution of the discharged waters.

Starting from September 3, 2002, significant changes in the temperature field occurred in Mamala Bay. It is seen from the temperature evolution shown in Fig. 5a that, during this period, the quasi-homogeneous layer descended to greater depths, sometimes occupying the entire water column from the surface to a depth of 60 m. The mean temperature of the layer significantly increased. During the first two days of the observations, the mean temperature of the layer was ~25.5°C and the amplitude of its fluctuations was 1–2°C. In the course of time, it increased up to 26.5–26.7°C on the average and the level of the fluctuations reached 3–4°C.

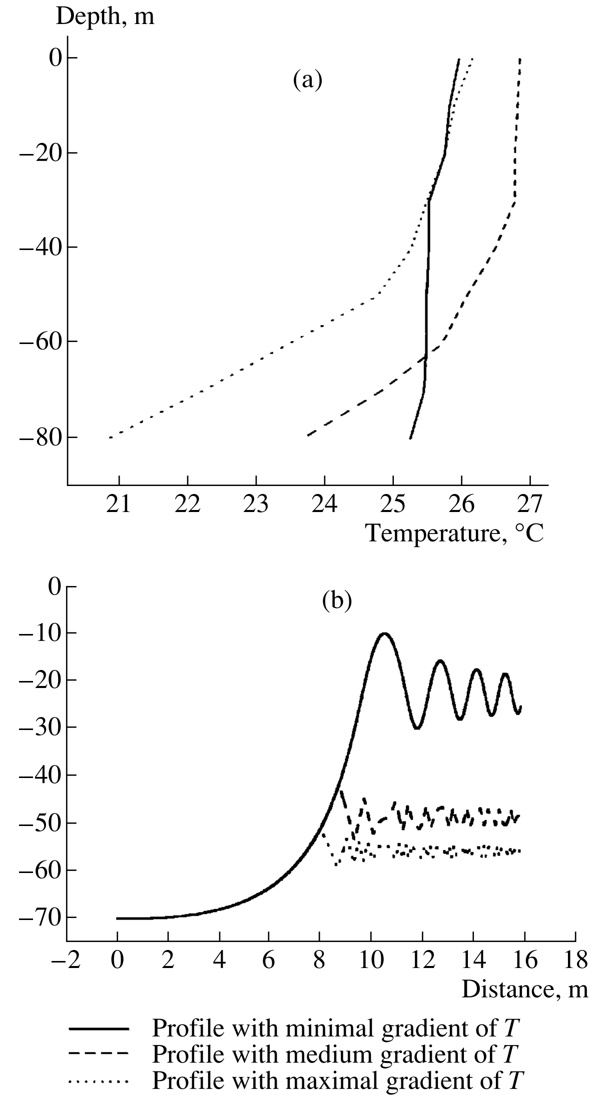

Fig. 4. Model estimates of the characteristics of the floating-up jets in the basin of Mamala Bay in the summer period of the year: (a) vertical profiles of the seawater temperature; (b) trajectories of the floating-up jets for the distributions of temperature with depth shown in Fig. 3a.

It is natural that the variations in the temperature properties influenced the stratification characteristics of the water masses. During the period from September 3 to September 6, 2002, the degree of the seawater stratification in Mamala Bay varied within wide limits sometimes reaching very low values.

Instantaneous vertical profiles of the seawater temperature in Mamala Bay are shown in Fig. 6c for three time moments during the period from September 3 to September 4, 2002; they point to significant variations in the stratification during this period.

The temporal evolution of the depth of the floating up and the initial dilution of the jet during the period from September 1 to September 6, 2002, are shown in Fig. 7. It is seen from this figure that, on the first two

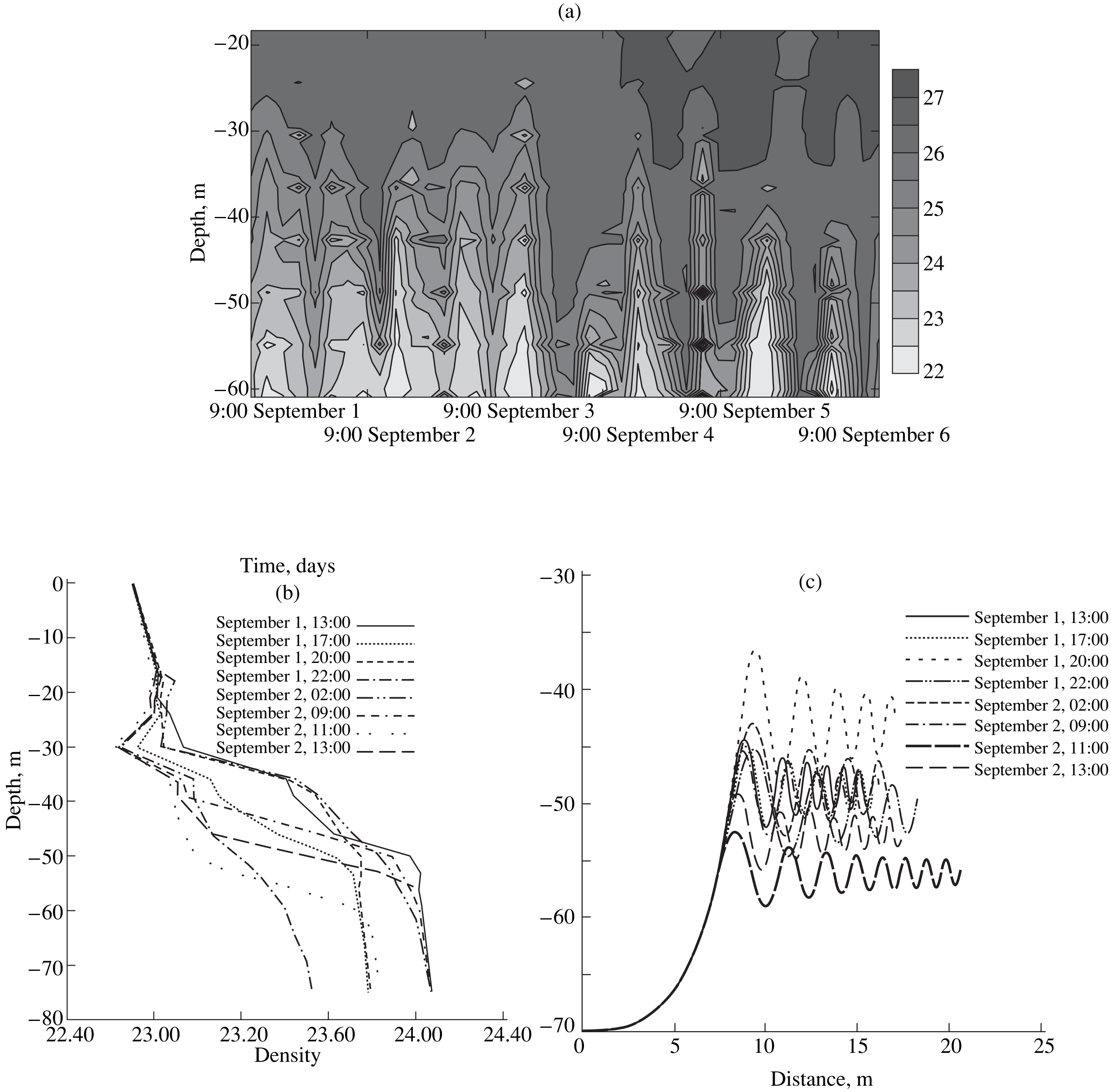

Fig. 5. Characteristics of the medium in Mamala Bay based on the results of field measurements: (a) time evolution of the seawater temperature in Mamala Bay at depths of 18–60 m during the period of September 1–6, 2002; (b) vertical profiles of the seawater density in Mamala Bay during the period from 13:00 on September 1 to 13:00 on September 2, 2002; (c) trajectories of propagation of the floating-up jets in Mamala Bay calculated from the data of the density profiles shown in Fig. 5b (c).

and a half days of the observations, the typical values of the height of the floating up z and the initial dilution of the jet D were within $z = -(42–32)$ m and $D = 150–250$, respectively.

After this, due to the increase in the amplitude of the vertical fluctuations of the thermocline and its deepening, the lower values of the float-up height and initial dilution of the jet decreased to $z_{min} = -50$ m and $D_{min} = 100$, and the maximum values of the float-up height and dilution increased to $z_{max} = 0$ m (surface) and $D_{min} = 960$.

An analysis of the data shown in Fig. 7 indicates that, in the period of observations from September 1 to September 6, 2002, the temperature stratification near the deep-water discharge varied within wide limits sometimes reaching very low values. Under these conditions (~15% of the all cases), strongly diluted (600–1000 times) discharge waters reached the surface (Fig. 7).

COMPARISON OF THE MODEL ESTIMATES OF THE JET PARAMETERS WITH THE DATA OF EXPERIMENTAL MEASUREMENTS

As was mentioned before, during the multidisciplinary experiments in Mamala Bay, the hydrophysical, optical, biological, and chemical parameters of the medium were measured using stationary moored stations; observations were also carried out from vessels including measurements with microstructure profilers [14, 15, 17, 19, 20]. The parameters of the jet propagation were determined on the basis of the results of these field measurements: the levels to which the discharge water floats up, the direction and velocity of the propagation, and the concentration of pollutants at different depths and distances from the diffuser.

A comparison of the parameters of the deep-water outfall discharges obtained on the basis of the experimental measurements with the results of the model calculations allows us to test whether the mathematical model applied is adequate and check the accuracy and reliability of the model estimates obtained.

The spatiotemporal distributions of different characteristics of the seawater were plotted on the basis of the measurements carried out using dropped and towed microstructure profilers (see Figs. 8a–8b). Profiles of the vertical distributions of the (a) turbidity, (b) salinity, and (c), temperature of the seawater plotted on the basis of the microstructure measurements near the diffuser on September 2, 2002, from 12:15 to 15:20 [14, 17, 19, 20] are shown in this figure.

It is clearly seen from these profiles that, during the period analyzed, the discharge waters ascended to a depth of 45 m.

The levels to which the jet of sewage waters floated up calculated using the model suggested in the period from 9:00 to 16:00 on September 2, 2002, are shown in Fig. 8d. It is seen from the figure that, during the period from 12:00 to 16:00, the model estimate of the mean level of the floating up is equal to ~44 m, which is in good agreement with the data of the experimental measurements (~45 m).

The results of processing the data of the hydrobiological measurements showed that, during the entire period of the observations, the discharged waters were located at depths above 40–50 m on the average. These average data also agree well with the results of the model calculations shown in Fig. 8d.

During the experiments from a research vessel on September 6, 2002, an anomalous spot at the sea surface was found near the diffuser. A photo of this surface anomaly taken by Professor C. Gibson at 14:48 is shown in Fig. 9a. The anomalous spot caused by the

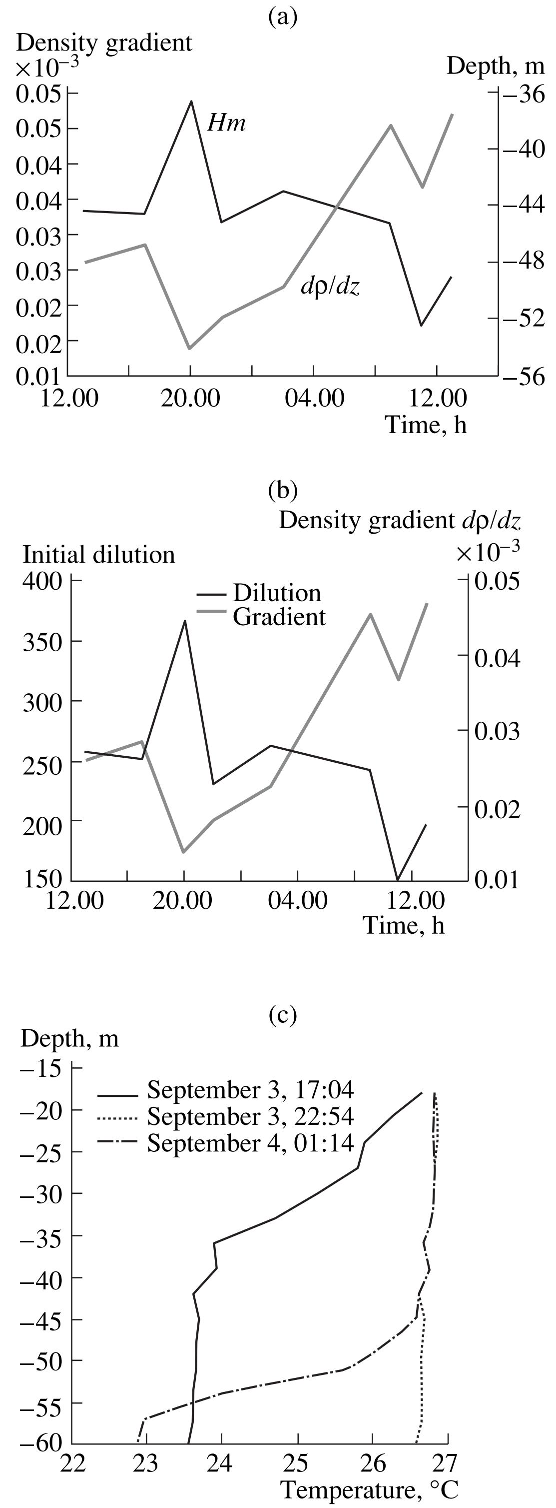

Fig. 6. Comparison of the parameters of jet propagation with the characteristics of the medium stratification: (a) time evolution of the level of float up of the jet *Hm* and the density gradient $d\rho/dz$; (b) time evolution of the initial dilution of the sewage waters and the density gradient $d\rho/dz$; (c) vertical profiles of the water temperature in Mamala Bay for three moments in the period from September 3 to September 4, 2002.

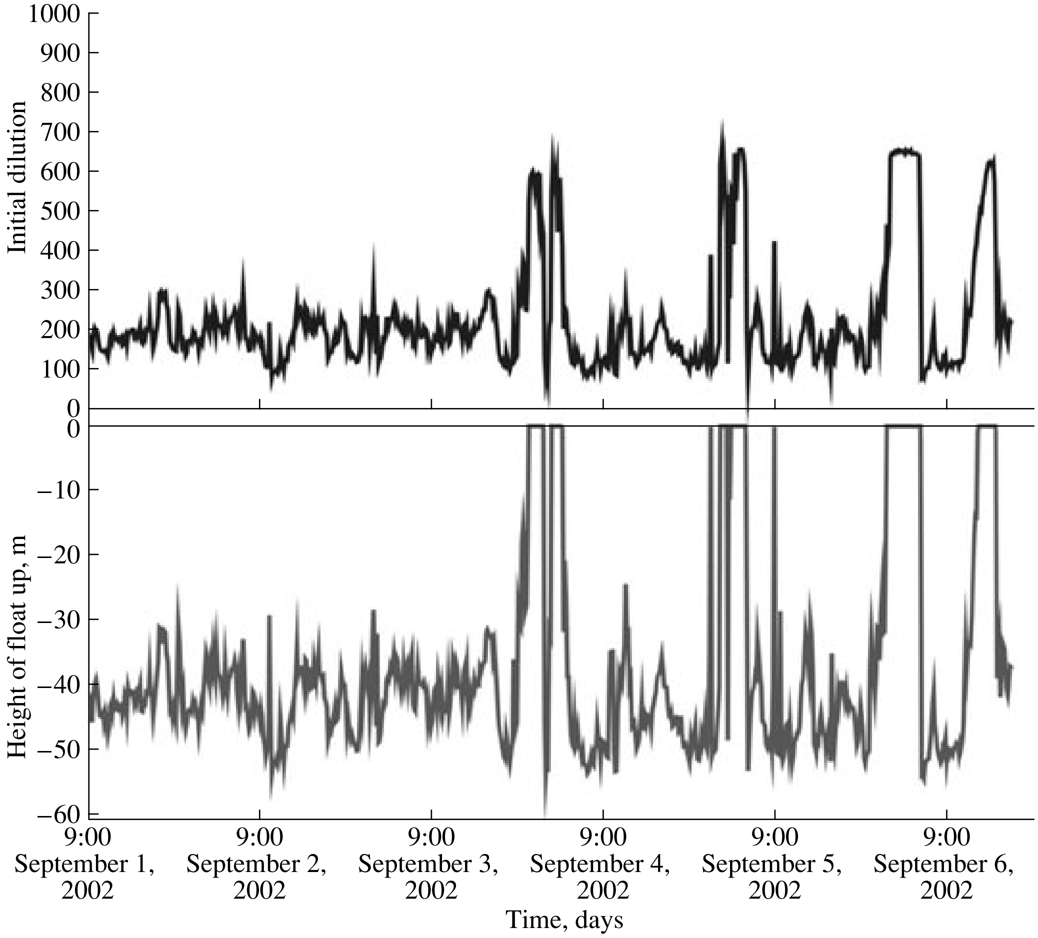

Fig. 7. Model estimates of the height of the float up and initial dilution in the period from September 1 to September 6, 2002.

surfacing of the discharged waters was distinguished at the sea surface by its color and the structure of the surface waves. Wave crests were observed at the boundaries of the spot. The diameter of the spot was equal to 80 m.

Vertical profiles of the temperature, salinity, density, and turbidity were measured at the center of the spot using a dropped microstructure profiler. They are presented in Fig. 9b [18]. It is seen from the profiles that the discharged waters that reached the surface were concentrated in the surface layer approximately 6 m thick.

The results of the model estimates of the float-up levels of the discharged waters on September 6, 2002, from 6:00 to 18:00 (local time) are presented in Fig. 9c. It is seen that, on the basis of the model data, the discharged waters reached the surface during the period from 13:00 to 15:00, which agrees well with the data of the experimental observations.

The good correlation between the model's estimates of the characteristics of the discharge waters with the results of the experimental measurements proves the adequate description of the mechanism of the propagation of deep turbulent jets by the applied mathematical model.

CONCLUSION

A theory has been developed for the propagation of turbulent jets of discharge waters in a stratified medium. On the basis of this theory, a computer model was developed that allows us to calculate the levels of floating up and dilution of the jets as well as to estimate the thicknesses of the layers in which the jets spread.

An analysis was performed of the potentialities of the methods of mathematical modeling for the study of characteristics of deep-water discharges in coastal regions.

The estimates of the parameters of a floating-up jet were performed using the developed mathematical model for deep sewage water discharge from Sand Island in the region of Mamala Bay (Hawaii Islands) at different versions of the discharge in different seasons of the year.

The estimates of the levels of floating up and initial dilution of the jet in a stratified medium were obtained

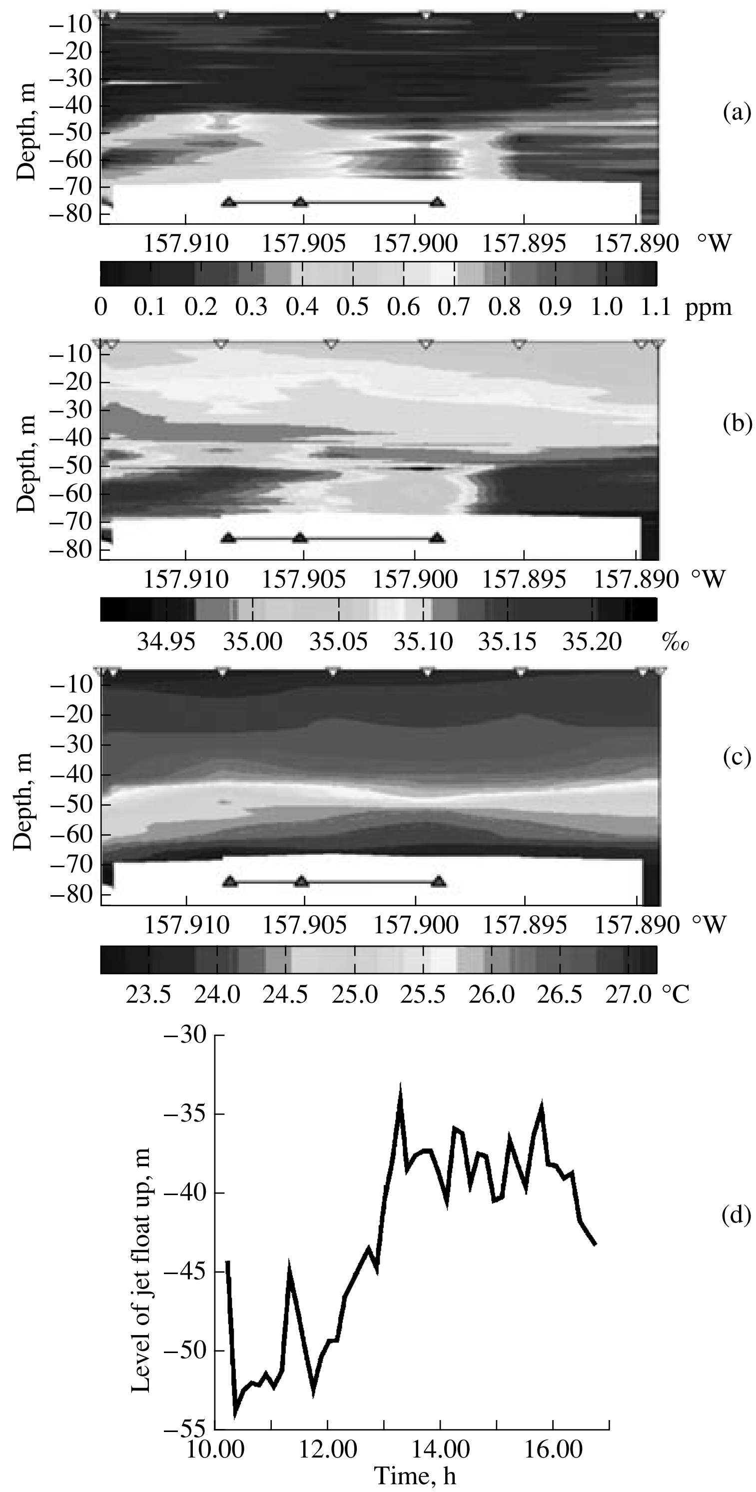

Fig. 8. Comparison of the model estimates of the parameters of the jet with the data of experimental measurements: vertical profiles of the (a) turbidity, (b) salinity, and (c) temperature on the basis of the measurements with an MSS profiler on September 2, 2002 during the period from 14:15 to 15:20; and (d) model estimates of the depth of the sewage water jet float up in the period from 9:00 to 17:00 on September 2, 2002.

from the results of the model calculations performed using the experimentally measured temperature and salinity profiles. It was shown that, during the period studied, the stratification in Mamala Bay was subjected

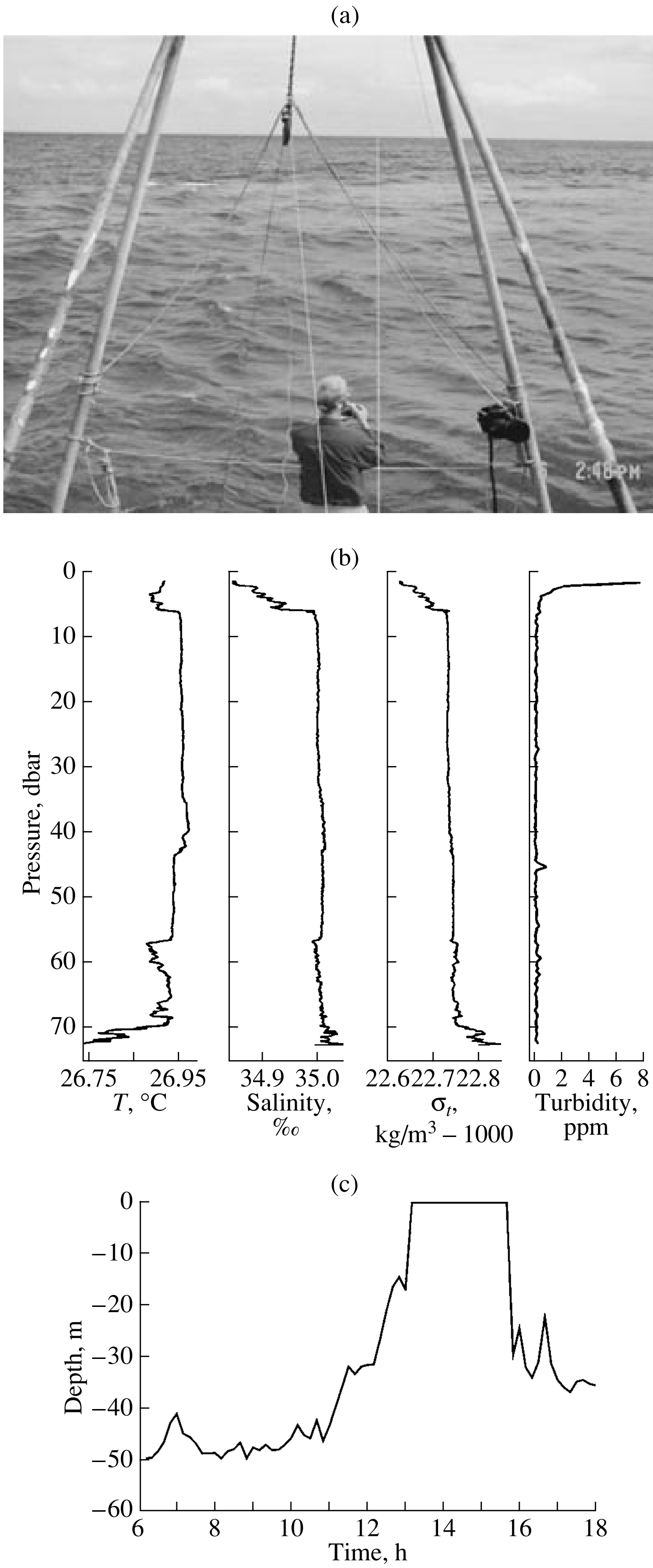

Fig. 9. Comparison of the model estimates of the parameters of the jet with the data of experimental measurements on September 6, 2002. (a) Photo of the surface anomaly caused by the deep-water discharge measured from a ship near the diffuser on September 6, 2002, at 14:48. (b) Vertical profiles of the temperature, salinity, density, and turbidity measured by a microstructure profiler on September 6, 2002, at the center of the anomalous patch (Fig. 9a). (c) Model estimates of the float-up depth of the sewage jet in the period from 6:00 to 18:00 on September 6, 2002 (c).

to significant fluctuations under the influence of tidal currents and related internal waves. The vertical oscillations of the thermocline under the influence of the tides were so large that, even in the summer period at a stable stratification, the sewage waters sometimes (~15% of the cases observed) reached the surface layer.

Comparisons of the model's estimates of the characteristics of the discharged water jet propagation with the data of experimental measurements showed good correlation, which points to the adequate description of the mechanism of the propagation of deep-water discharges by the applied mathematical model.

The results of the mathematical modeling of the propagation of a discharge water jet in seawater demonstrate the efficiency of applying the suggested model for investigating deep discharges in coastal regions.

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